The Disintegration (or not) of the Nonlinear Internal Tide

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LONG-TERM GOALS

This research is aimed at studying the underlying dynamics of, and identifying the conditions that control, the disintegration of the internal tide into large-amplitude internal solitary-like waves.

OBJECTIVES

The objectives are to use a combination of theoretical and numerical models to study the evolution of the internal tide and its possible disintegration into internal solitary waves. A central aspect of this work is to explore the role of rotation in the process. Rotation permits the presence of periodic, nonlinear inertia-gravity waves (i.e., the tide) that can act as attractors and arrest the steepening of the internal tide, and hence affect the production of the shorter solitary-like waves (Gerkema, 1996). In light of recent observations of strongly nonlinear internal solitary waves in the South China Sea (e.g. Duda *et al*, 2004; Ramp *et al*., 2004; Zhao and Alford, 2006) and numerous other locations, an important objective is to allow for fully nonlinear waves. A further objective is to test these theories and models with observations obtained from the NLIWI South China Sea DRI in order to improve the ability to predict the arrival of large-amplitude internal solitary waves.

APPROACH

The approach combines theoretical wave evolution models and numerical solutions of these models and solutions of the full Navier-Stokes equations. The theoretical models require some simplifications including restriction to two-layer flows and one-dimensional propagation. The presence of rotation requires flow in the direction transverse to the propagation; however, variations of properties in this direction are ignored. The theory is an extension of the fully-nonlinear, weakly non-hydrostatic internal wave theory of Miyata (1988) and Choi and Camassa (1999) to include rotation. In most cases these model equations are solved numerically using modern, high-order schemes, though some analytical progress has been made. This theory will be complemented using a 2.5-dimensional Navier-Stokes numerical model that permits continuous stratification and eliminates restrictions associated with the long-wave assumption in the theory. Variable topography can be included in both the theory and the numerical models.

The theoretical and modeling results are being compared against PIES observations by David Farmer (URI) and other NILIWI investigators of the low-mode internal tide evolution across the South China Sea from just west of Luzon Strait to the Chinese shelf.

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WORK COMPLETED

The Miyata/Choi-Camassa (MCC) theory has been extended to include rotation (2.5 dimension MCC-f model) and a numerical scheme has been developed and tested. In Helfrich (2007a) the effects of weak background rotation on the evolution of large-amplitude internal solitary waves has been studied. This avenue of investigation is continuing in collaboration with Roger Grimshaw at Loughborough University with an investigation of the nonlinear wave packet dynamics (Grimshaw and Helfrich, in prep).

The second completed part of this project is a study of the nonlinear internal tide solutions of the two-layer MCC-f equations (in the hydrostatic limit) and the disintegration of an initially sinusoidal internal tide into remnant internal tide and high-frequency solitary-like waves (Helfrich and Grimshaw, 2007). A second paper extends the hydrostatic, fully-nonlinear nonlinear internal tide theory to include continuous stratification (Helfrich 2007b). The role of nonhydrostatic effects and the production of solitary-like waves from the disintegration of an initial sinusoidal internal tide are again investigated.

In another paper a theory for steady gravity currents in continuously stratified fluids is derived (White and Helfrich, 2007). The gravity current theory is used to determine when either a steady gravity current or a gravity current that generates unsteady nonlinear internal waves that propagate ahead of the gravity current are to be expected. Numerical solutions (2-dimensional Navier-Stokes equations) of the initial value dam-break problem are used to test the theoretical results.

RESULTS

From weakly nonlinear theory it is known that steadily propogating solitary wave solutions do not exist in the presence of rotation. An initial solitary wave will decay in finite time due to the radiation of longer inertia-gravity waves (Grimshaw *et al*., 1998). In a few numerical solutions the weakly nonlinear regime Grimshaw *et al*. (1998) noted that the radiation process could lead to the periodic re-emergence of a solitary wave through steepening of a radiated inertia-gravity wave. This work has been extended into the fully-nonlinear regime through solutions of the MCC-f equations (Helfrich, 2007a). The re-emergence process not only persists to large amplitude, it is enhanced. The new solutions indicate that the process is related to the generation of localized envelope, or packet, through which the solitary waves propagate, decaying at the leading edge and emerging at the trailing edge. An example of a numerical solution is shown in Figure 1. Depending on the parameters, as much as 50% of the energy in the initial wave can be retained in the packet. Current work is underway to address the dynamics of the packets (Grimshaw and Helfrich, in prep.).

Earlier work on the role of rotation on the evolution of an internal tide had been restricted to weakly nonlinear waves (e.g., Gerkema, 1996; Holloway *et al.,* 1999). Gerkema (1996) found that with rotation the fission of the tide into solitary wave packets could be inhibited. This was due to the presence of rotational dispersion, which could balance the nonlinearity to give hydrostatic nonlinear tide (inertia-gravity) solutions that prevented further steepening of the tide, and thus the emergence of shorter solitary waves.

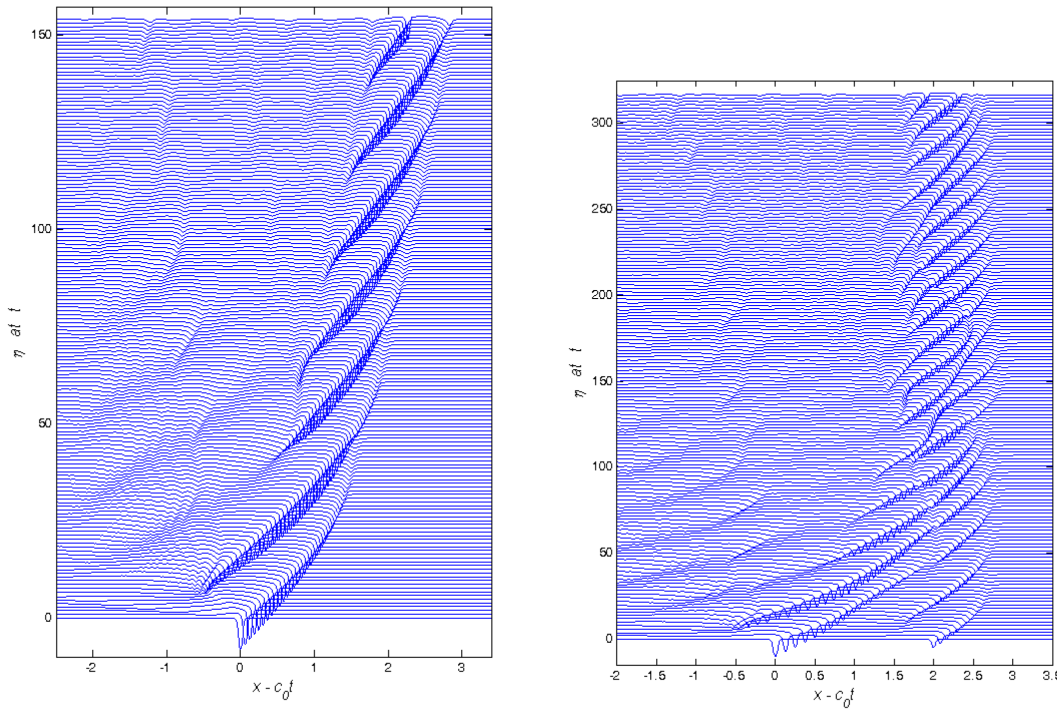

Figure 1. Decay and re-emergence of a large-amplitude internal solitary wave in the presence of rotation. The LEFT panel shows the interface in a frame moving with the linear long wave phase speed, c_0. The upper layer depth is 0.25 of the total depth, the initial wave amplitude is 0.2, and the nonhydrostatic parameter $\beta^{1/2}$=0.02. The wave initially decays only to re-emerge from the steepening of the radiated inertia-gravity wave. The process repeats until a coherent packet of solitary-like waves develops. The packet is not perfectly localized, but the energy contained in the packet decays very slowly. The RIGHT panel shows and interaction between two packets that leads to packet merger. The initial solitary waves amplitudes are 0.2 and 0.1 and the other parameters are the same as the left panel.

Since observations frequently show both tides and waves with amplitudes beyond the restrictions of weakly nonlinear theory, the disintegration of an initial long internal tide has been studied using the fully-nonlinear, two-layer MCC-f theory (Helfrich and Grimshaw, 2007). In the hydrostatic limit, the governing equations have periodic, nonlinear inertia-gravity solutions that are explored as models of the nonlinear internal tide. These long waves are robust to weak nonhydrostatic effects. Numerical solutions show that the disintegration of an initial sinusoidal linear internal tide is closely linked to the presence of these nonlinear waves (see Figure 2). The initial tide steepens due to nonlinearity and sheds energy into short solitary-like waves. The disintegration is halted as the longwave part of the solution settles onto one of the nonlinear hydrostatic solutions, with the short solitary waves superimposed. The degree of disintegration is a function of initial amplitude of the tide and the properties of the underlying nonlinear hydrostatic solutions, which, depending on stratification and tidal frequency, exist only for a finite range of amplitudes (or energies). There is a lower threshold below which no short solitary waves are produced. However, for initial amplitudes above another threshold, given approximately by the energy of the limiting nonlinear hydrostatic inertia-gravity wave, most of the initial tidal energy goes into solitary waves. These ideas have been applied to the South China Sea where it is shown that

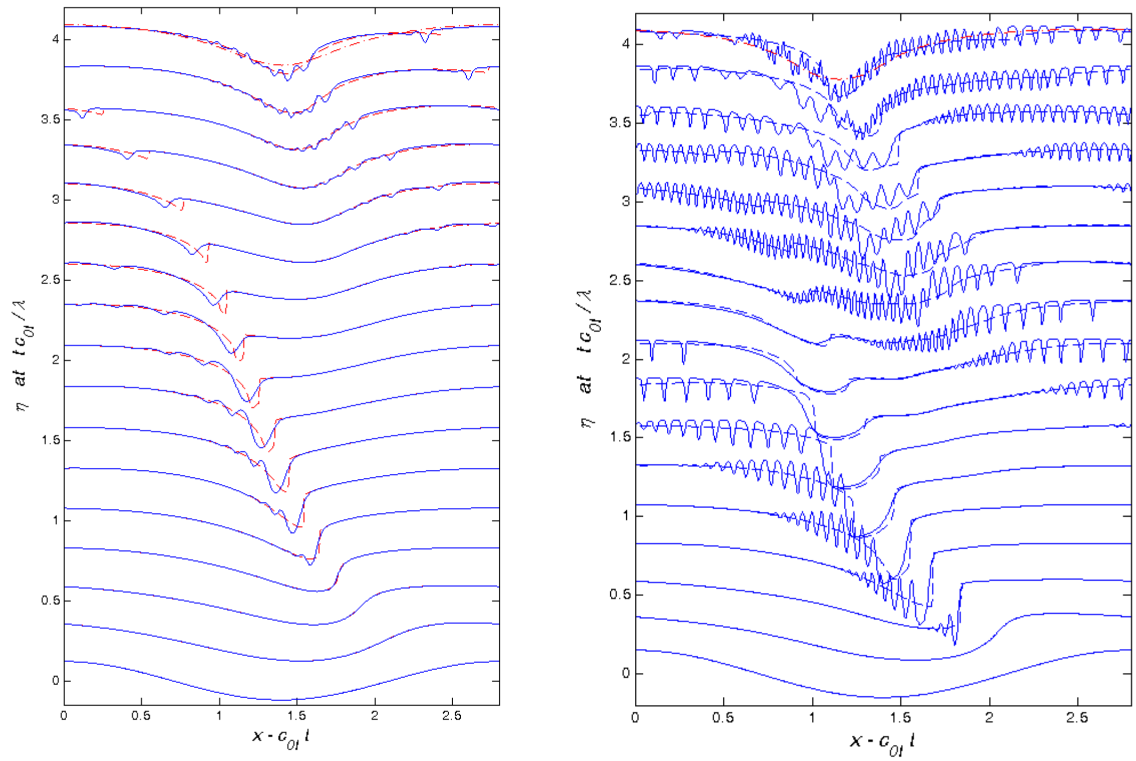

Figure 2. Evolution of an initially sinusoidal internal tide. LEFT: The figure shows the interface over several tidal periods of an initial tide with amplitude a=0.06. The solid blue line is the solution to the MCC-f equations and the dashed red line shows a solution to the hydrostatic shallow water equations (MCC-f in the hydrostatic limit). The red dash-dot line is the theoretical hydrostatic tide with a same energy as the hydrostatic solution. RIGHT: An identical set of computations except the initial tide amplitude a=0.1. The production of short solitary-like waves increases with amplitude, but in both cases an underlying tide remains.

the diurnal tide is much less likely to disintegrate into nonlinear solitary-like waves than is the semidiurnal tide. This is in qualitative agreement with the observations of Zhao and Alford (2006).

The MCC-f theory is informative; however, in the present form, it is restricted to two-layers. This limitation has been lifted in Helfrich (2007b) where the nonlinear tide theory is extended to allow continuous stratification. Figure 3 shows the limiting (maximum amplitude) diurnal internal tide solution computed for the stratification from the northern South China Sea. The tide has a corner shape with maximum isopycnal displacements of 93 m. It was not possible to find nonlinear tide solutions at the semidiurnal frequency. As a result, in initial sinusoidal internal tide at the semidiurnal frequency will disintegrate almost completely into short nonlinear solitary-like waves, which the diurnal tide is more likely to remain intact and inhibit the production of NLIWs.

Recent observations of NLIW generation by the Columbia River plume (Nash and Moum, 2005) indicated that large NLIWs can be generated by the propagation of a gravity current into a stratified fluid. Their observations suggested that the waves are released from the gravity currents as the gravity

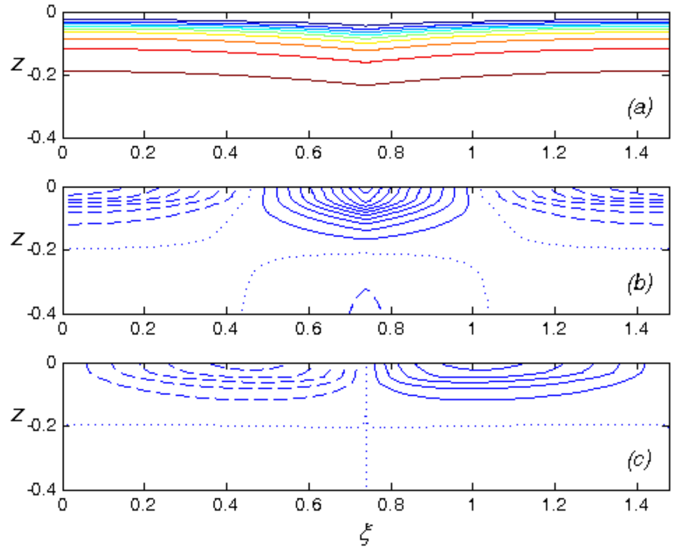

Figure 3. Fully nonlinear, hydrostatic tide solution for the diurnal tide in the South China Sea. The density field is shown in (a), the x- velocity field in (b), and the transverse, y- velocityin (c). Here ξ = x-ct, where c is the wave phase speed. In (a) and (b) the solid (dashed) contours indicate positive (negative) velocities.This solution is the maximum allowable amplitude and in dimensional variables the wave is 363 km and the maximum isopycnal displacement is 93 m.

current front speed passes from super- to sub-critical (compared to the linear long wave phase speed). A theory for gravity currents propagating into a stratified fluid with a general density profile has been derived (White and Helfrich, 2007). The theory predicts steady gravity currents with constant front speed over a range of current thicknesses and speeds. The solutions have two limits: an energy conserving solution (the conjugate state, or limiting amplitude solitary wave with a trapped core) and a condition of incipient convective instability. Navier-Stokes simulations of a lock release initial value problem show that in some cases internal waves are generated and propagate ahead of the gravity (see Figure 4). The conditions leasing to internal wave generation correspond to the range in which the steady gravity current theory fails. Thus the theory predicts the occurance of either wave-generating or steady gravity solutions to the dam break problem. In contrast to the Nash and Moum observations, this study shows that wave generation does not depend on deceleration of the gravity current from super- to sub-critical conditions. Deceleration of the gravity currents simply hastens the wave release.

IMPACT/APPLICATIONS

The ubiquitous nature of large amplitude internal solitary waves in the world's coastal oceans and marginal seas is clear from observations. These waves are can have significant effects on coastal mixing through breaking as they propagate and shoal, and they may also lead to substantial horizontal mass transport. Since the waves are frequently generated through the radiation of an internal tide by barotropic tidal flow over localized topography (as is apparently the case at in the Luzon Strait), this work will help understand what fraction of the energy put in at the tidal frequency ends up as internal solitary waves, the space and time scales for that transformation, and the characteristics of the resulting solitary-like waves.

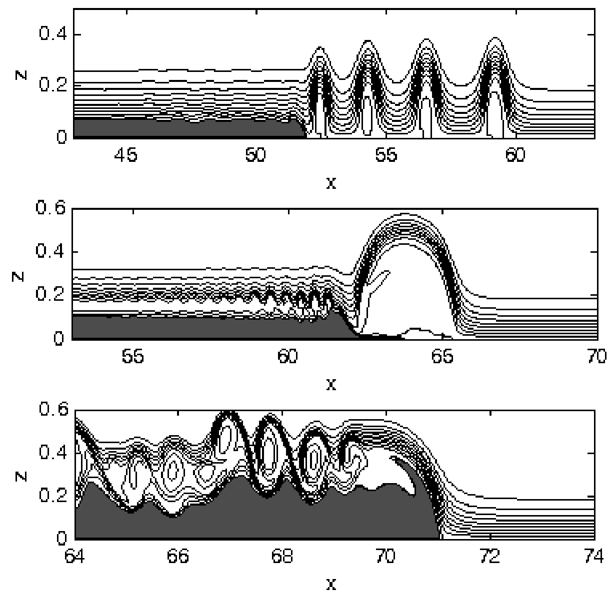

Figure 4. Numerical solutions of the Navier-Stokes equations showing the leading portion of a gravity current (gray shading) and upstream nonlinear internal waves. The gravity current is produced by a dam-break far to the left in each frame. The depth of the dammed fluid increases from the TOP to the BOTTOM panel. In the TOP panel the gravity current generates an upstream group of nonlinear internal waves that outrun the gravity current. The MIDDLE panel shows a case where the gravity current generates one very large wave that is phase locked to the front of the gravity current. In the BOTTOM panel no upstream waves are produced and the gravity current properties (speed, height and ambient density and velocity fields) are in good agreement with the theoretically predicted dissipationless (conjugate state) gravity current.

The radiation decay and reemergence of internal solitary waves offers a possible explanation of why it is frequently difficult to track internal solitary waves. In the presence of rotation, and over the appropriate space and time scales, the individual waves are ephemeral, though the wave energy remains in coherent packets.

The study of the coupling of gravity currents and NLIWs involves a less well understood, but potentially important, NLIW generation mechanism.

This basic theoretical and numerical work will be directly applicable to the analysis and interpretation of the NLIWI South China Sea observations. To this end, current efforts are aimed at direct comparisons of the MCC-f and continuously stratified models with the PIES observations by David Farmer (URI) of the evolution of the internal tide and NLIWs across the northeastern South China Sea.

RELATED PROJECTS

This work is directly related to the ONR NLIWI South China Sea DRI.

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